Maple 2018.2 Integration Test Results
on the problems in "7 Inverse hyperbolic functions/7.6 Inverse hyperbolic cosecant"

Test results for the 48 problems in "7.6.1 u (a+b arccsch(c x))^n.txt"

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b\,\mathrm{arccsch}(c\,x)}{x^3}\, dx$$

Optimal(type 3, 45 leaves, 4 steps):

$$-\frac{b\,c^2\,\mathrm{arccsch}(c\,x)}{4} + \frac{-a - b\,\mathrm{arccsch}(c\,x)}{2\,x^2} + \frac{b\,c\,\sqrt{1 + \frac{1}{c^2\,x^2}}}{4\,x}$$

Result(type 3, 99 leaves):

$$c^2\left(-\frac{a}{2\,c^2\,x^2} + b\left(-\frac{\mathrm{arccsch}(c\,x)}{2\,c^2\,x^2} - \frac{\sqrt{c^2\,x^2+1}\left(\mathrm{arctanh}\left(\frac{1}{\sqrt{c^2\,x^2+1}}\right)c^2\,x^2 - \sqrt{c^2\,x^2+1}\right)}{4\,\sqrt{\frac{c^2\,x^2+1}{c^2\,x^2}}\,c^3\,x^3}\right)\right)$$

Problem 5: Unable to integrate problem.

$$\int x^3\,(a + b\,\mathrm{arccsch}(c\,x))^2\, dx$$

Optimal(type 3, 91 leaves, 5 steps):

$$\frac{b^2\,x^2}{12\,c^2} + \frac{x^4\,(a + b\,\mathrm{arccsch}(c\,x))^2}{4} - \frac{b^2\,\ln(x)}{3\,c^4} - \frac{b\,x\,(a + b\,\mathrm{arccsch}(c\,x))\,\sqrt{1 + \frac{1}{c^2\,x^2}}}{3\,c^3} + \frac{b\,x^3\,(a + b\,\mathrm{arccsch}(c\,x))\,\sqrt{1 + \frac{1}{c^2\,x^2}}}{6\,c}$$

Result(type 8, 16 leaves):

$$\int x^3\,(a + b\,\mathrm{arccsch}(c\,x))^2\, dx$$

Problem 6: Unable to integrate problem.

$$\int x^2\,(a + b\,\mathrm{arccsch}(c\,x))^2\, dx$$

Optimal(type 4, 148 leaves, 8 steps):

$$\frac{b^2\,x}{3\,c^2} + \frac{x^3\,(a + b\,\mathrm{arccsch}(c\,x))^2}{3} - \frac{2\,b\,(a + b\,\mathrm{arccsch}(c\,x))\,\mathrm{arctanh}\left(\frac{1}{c\,x} + \sqrt{1 + \frac{1}{c^2\,x^2}}\right)}{3\,c^3} - \frac{b^2\,\mathrm{polylog}\left(2, -\frac{1}{c\,x} - \sqrt{1 + \frac{1}{c^2\,x^2}}\right)}{3\,c^3}$$

$$+\frac{b^2 \operatorname{polylog}\left(2, \frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)}{3 c^3}+\frac{b x^2 (a+b \operatorname{arccsch}(c x)) \sqrt{1+\frac{1}{c^2 x^2}}}{3 c}$$

Result(type 8, 16 leaves):

$$\int x^2 (a+b \operatorname{arccsch}(c x))^2 \, dx$$

Problem 7: Unable to integrate problem.

$$\int x (a+b \operatorname{arccsch}(c x))^2 \, dx$$

Optimal(type 3, 50 leaves, 4 steps):

$$\frac{x^2 (a+b \operatorname{arccsch}(c x))^2}{2}+\frac{b^2 \ln(x)}{c^2}+\frac{b x (a+b \operatorname{arccsch}(c x)) \sqrt{1+\frac{1}{c^2 x^2}}}{c}$$

Result(type 8, 14 leaves):

$$\int x (a+b \operatorname{arccsch}(c x))^2 \, dx$$

Problem 8: Unable to integrate problem.

$$\int (a+b \operatorname{arccsch}(c x))^2 \, dx$$

Optimal(type 4, 108 leaves, 7 steps):

$$x (a+b \operatorname{arccsch}(c x))^2+\frac{4 b (a+b \operatorname{arccsch}(c x)) \operatorname{arctanh}\left(\frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)}{c}+\frac{2 b^2 \operatorname{polylog}\left(2, -\frac{1}{c x}-\sqrt{1+\frac{1}{c^2 x^2}}\right)}{c}$$
$$-\frac{2 b^2 \operatorname{polylog}\left(2, \frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)}{c}$$

Result(type 8, 12 leaves):

$$\int (a+b \operatorname{arccsch}(c x))^2 \, dx$$

Problem 9: Unable to integrate problem.

$$\int \frac{(a+b \operatorname{arccsch}(c x))^2}{x} \, dx$$

Optimal(type 4, 116 leaves, 6 steps):

$$\frac{(a+b\,\mathrm{arccsch}(c\,x))^3}{3\,b}-(a+b\,\mathrm{arccsch}(c\,x))^2\ln\left(1-\left(\frac{1}{c\,x}+\sqrt{1+\frac{1}{c^2\,x^2}}\right)^2\right)-b\,(a+b\,\mathrm{arccsch}(c\,x))\,\mathrm{polylog}\left(2,\left(\frac{1}{c\,x}+\sqrt{1+\frac{1}{c^2\,x^2}}\right)^2\right)$$
$$+\frac{b^2\,\mathrm{polylog}\left(3,\left(\frac{1}{c\,x}+\sqrt{1+\frac{1}{c^2\,x^2}}\right)^2\right)}{2}$$

Result(type 8, 16 leaves):

$$\int\frac{(a+b\,\mathrm{arccsch}(c\,x))^2}{x}\,\mathrm{d}x$$

Problem 10: Unable to integrate problem.

$$\int\frac{(a+b\,\mathrm{arccsch}(c\,x))^2}{x^5}\,\mathrm{d}x$$

Optimal(type 3, 114 leaves, 5 steps):

$$-\frac{b^2}{32\,x^4}+\frac{3\,b^2\,c^2}{32\,x^2}+\frac{3\,a\,b\,c^4\,\mathrm{arccsch}(c\,x)}{16}+\frac{3\,b^2\,c^4\,\mathrm{arccsch}(c\,x)^2}{32}-\frac{(a+b\,\mathrm{arccsch}(c\,x))^2}{4\,x^4}+\frac{b\,c\,(a+b\,\mathrm{arccsch}(c\,x))\,\sqrt{1+\frac{1}{c^2\,x^2}}}{8\,x^3}$$
$$-\frac{3\,b\,c^3\,(a+b\,\mathrm{arccsch}(c\,x))\,\sqrt{1+\frac{1}{c^2\,x^2}}}{16\,x}$$

Result(type 8, 16 leaves):

$$\int\frac{(a+b\,\mathrm{arccsch}(c\,x))^2}{x^5}\,\mathrm{d}x$$

Problem 11: Unable to integrate problem.

$$\int\frac{(a+b\,\mathrm{arccsch}(c\,x))^3}{x^2}\,\mathrm{d}x$$

Optimal(type 3, 74 leaves, 5 steps):

$$-\frac{6\,b^2\,(a+b\,\mathrm{arccsch}(c\,x))}{x}-\frac{(a+b\,\mathrm{arccsch}(c\,x))^3}{x}+6\,b^3\,c\,\sqrt{1+\frac{1}{c^2\,x^2}}+3\,b\,c\,(a+b\,\mathrm{arccsch}(c\,x))^2\,\sqrt{1+\frac{1}{c^2\,x^2}}$$

Result(type 8, 16 leaves):

$$\int\frac{(a+b\,\mathrm{arccsch}(c\,x))^3}{x^2}\,\mathrm{d}x$$

Problem 14: Unable to integrate problem.

$$\int (d x)^m (a + b \operatorname{arccsch}(c x)) \, dx$$

Optimal(type 5, 63 leaves, 3 steps):

$$\frac{(d x)^{1+m} (a + b \operatorname{arccsch}(c x))}{d (1+m)} + \frac{b (d x)^m \operatorname{hypergeom}\left(\left[\frac{1}{2}, -\frac{m}{2}\right], \left[1 - \frac{m}{2}\right], -\frac{1}{c^2 x^2}\right)}{c m (1+m)}$$

Result(type 8, 16 leaves):

$$\int (d x)^m (a + b \operatorname{arccsch}(c x)) \, dx$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \frac{a + b \operatorname{arccsch}(c x)}{(e x + d)^2} \, dx$$

Optimal(type 3, 94 leaves, 7 steps):

$$\frac{b \operatorname{arccsch}(c x)}{d e} + \frac{-a - b \operatorname{arccsch}(c x)}{e (e x + d)} + \frac{b \operatorname{arctanh}\left(\frac{c^2 d - \frac{e}{x}}{c \sqrt{c^2 d^2 + e^2} \sqrt{1 + \frac{1}{c^2 x^2}}}\right)}{d \sqrt{c^2 d^2 + e^2}}$$

Result(type 3, 207 leaves):

$$-\frac{c a}{(c e x + d c) e} - \frac{c b \operatorname{arccsch}(c x)}{(c e x + d c) e} + \frac{b \sqrt{c^2 x^2 + 1} \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2 x^2 + 1}}\right)}{c e \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} x d} - \frac{b \sqrt{c^2 x^2 + 1} \ln\left(\frac{2 \left(\sqrt{\frac{c^2 d^2 + e^2}{e^2}} \sqrt{c^2 x^2 + 1} e - d c^2 x + e\right)}{c e x + d c}\right)}{c e \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}} x d \sqrt{\frac{c^2 d^2 + e^2}{e^2}}}$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int (e x + d)^{3/2} (a + b \operatorname{arccsch}(c x)) \, dx$$

Optimal(type 4, 415 leaves, 22 steps):

$$\frac{2 (e x + d)^{5/2} (a + b \operatorname{arccsch}(c x))}{5 e} + \frac{4 b e (c^2 x^2 + 1) \sqrt{e x + d}}{15 c^3 x \sqrt{1 + \frac{1}{c^2 x^2}}} + \frac{28 b c d \operatorname{EllipticE}\left(\frac{\sqrt{1 - x \sqrt{-c^2}} \sqrt{2}}{2}, \sqrt{-\frac{2 e \sqrt{-c^2}}{c^2 d - e \sqrt{-c^2}}}\right) \sqrt{e x + d} \sqrt{c^2 x^2 + 1}}{15 (-c^2)^{3/2} x \sqrt{1 + \frac{1}{c^2 x^2}} \sqrt{\frac{e x + d}{d + \frac{e}{\sqrt{-c^2}}}}}$$

$$-\frac{4 b c \left(2 c^2 d^2 - e^2\right) \mathrm{EllipticF}\left(\frac{\sqrt{1 - x\sqrt{-c^2}}\,\sqrt{2}}{2}, \sqrt{-\frac{2 e\sqrt{-c^2}}{c^2 d - e\sqrt{-c^2}}}\right) \sqrt{c^2 x^2 + 1}\, \sqrt{\frac{e x + d}{d + \frac{e}{\sqrt{-c^2}}}}}{15 \left(-c^2\right)^{5/2} x \sqrt{1 + \frac{1}{c^2 x^2}}\, \sqrt{e x + d}}$$

$$-\frac{4 b d^3 \mathrm{EllipticPi}\left(\frac{\sqrt{1 - x\sqrt{-c^2}}\,\sqrt{2}}{2}, 2, \sqrt{2}\,\sqrt{\frac{e}{d\sqrt{-c^2} + e}}\right) \sqrt{c^2 x^2 + 1}\, \sqrt{\frac{(e x + d)\sqrt{-c^2}}{d\sqrt{-c^2} + e}}}{5 c e x \sqrt{1 + \frac{1}{c^2 x^2}}\, \sqrt{e x + d}}$$

Result(type 4, 1938 leaves):

$$\frac{1}{e}\left(2\left(\frac{(e x + d)^{5/2} a}{5} + b\left(\frac{(e x + d)^{5/2} \operatorname{arccsch}(c x)}{5} + \left(2\left(\right.\right.\right.\right.$$

$$-I\sqrt{-\frac{I(e x + d) c e + (e x + d) c^2 d - c^2 d^2 - e^2}{c^2 d^2 + e^2}}\,\sqrt{\frac{I(e x + d) c e - (e x + d) c^2 d + c^2 d^2 + e^2}{c^2 d^2 + e^2}}\,\mathrm{EllipticF}\left(\sqrt{e x + d}\,\sqrt{\frac{(I e + d c) c}{c^2 d^2 + e^2}},\right.$$

$$\left.\sqrt{-\frac{2 I c d e - c^2 d^2 + e^2}{c^2 d^2 + e^2}}\right) e^3 - \sqrt{\frac{(I e + d c) c}{c^2 d^2 + e^2}}\,(e x + d)^{5/2} c^3 d - 2 I\sqrt{\frac{(I e + d c) c}{c^2 d^2 + e^2}}\,(e x + d)^{3/2} c^2 d e$$

$$-9\sqrt{-\frac{I(e x + d) c e + (e x + d) c^2 d - c^2 d^2 - e^2}{c^2 d^2 + e^2}}\,\sqrt{\frac{I(e x + d) c e - (e x + d) c^2 d + c^2 d^2 + e^2}{c^2 d^2 + e^2}}\,\mathrm{EllipticF}\left(\sqrt{e x + d}\,\sqrt{\frac{(I e + d c) c}{c^2 d^2 + e^2}},\right.$$

$$\left.\sqrt{-\frac{2 I c d e - c^2 d^2 + e^2}{c^2 d^2 + e^2}}\right) c^3 d^3$$

$$+7\sqrt{-\frac{I(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{I(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}}\,\mathrm{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}},\right.$$

$$\left.\sqrt{-\frac{2Icde-c^2d^2+e^2}{c^2d^2+e^2}}\right)c^3d^3+I\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}\,(ex+d)^{5/2}c^2e$$

$$+3\sqrt{-\frac{I(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{I(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}}\,\mathrm{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}},\right.$$

$$\left.\frac{c^2d^2+e^2}{(Ie+dc)cd},\frac{\sqrt{-\frac{(Ie-dc)c}{c^2d^2+e^2}}}{\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}}\right)c^3d^3$$

$$+2I\sqrt{-\frac{I(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{I(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}},\right.$$

$$\left.\sqrt{-\frac{2Icde-c^2d^2+e^2}{c^2d^2+e^2}}\right)c^2d^2e+2\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}\,(ex+d)^{3/2}c^3d^2+I\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}\sqrt{ex+d}\,e^3-\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}\sqrt{ex+d}\,c^3d^3$$

$$+I\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}\sqrt{ex+d}\,c^2d^2e$$

$$-6\sqrt{-\frac{I(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{I(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}}\,\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}},\right.$$

$$\sqrt{-\frac{2 I c d e - c^2 d^2 + e^2}{c^2 d^2 + e^2}}\Bigg) c d e^2$$

$$+ 7 \sqrt{-\frac{I (e x + d) c e + (e x + d) c^2 d - c^2 d^2 - e^2}{c^2 d^2 + e^2}} \sqrt{\frac{I (e x + d) c e - (e x + d) c^2 d + c^2 d^2 + e^2}{c^2 d^2 + e^2}} \operatorname{EllipticE}\Bigg(\sqrt{e x + d} \sqrt{\frac{(I e + d c) c}{c^2 d^2 + e^2}},$$

$$\sqrt{-\frac{2 I c d e - c^2 d^2 + e^2}{c^2 d^2 + e^2}}\Bigg) c d e^2$$

$$- 3 I \sqrt{-\frac{I (e x + d) c e + (e x + d) c^2 d - c^2 d^2 - e^2}{c^2 d^2 + e^2}} \sqrt{\frac{I (e x + d) c e - (e x + d) c^2 d + c^2 d^2 + e^2}{c^2 d^2 + e^2}} \operatorname{EllipticPi}\Bigg(\sqrt{e x + d} \sqrt{\frac{(I e + d c) c}{c^2 d^2 + e^2}},$$

$$\frac{c^2 d^2 + e^2}{(I e + d c) c d}, \frac{\sqrt{-\frac{(I e - d c) c}{c^2 d^2 + e^2}}}{\sqrt{\frac{(I e + d c) c}{c^2 d^2 + e^2}}}\Bigg) c^2 d^2 e - \sqrt{\frac{(I e + d c) c}{c^2 d^2 + e^2}} \sqrt{e x + d}\, c d e^2\Bigg)\Bigg)\Bigg/$$

$$\left(15 c^3 \sqrt{\frac{(e x + d)^2 c^2 - 2 (e x + d) c^2 d + c^2 d^2 + e^2}{c^2 x^2 e^2}}\, x \sqrt{\frac{(I e + d c) c}{c^2 d^2 + e^2}} (I e - d c)\right)\Bigg)\Bigg)\Bigg)$$

Problem 19: Result more than twice size of optimal antiderivative.

$$\int \frac{x (a + b \operatorname{arccsch}(c x))}{\sqrt{e x + d}}\, dx$$

Optimal(type 4, 407 leaves, 14 steps):

$$\frac{2 (e x + d)^{3/2} (a + b \operatorname{arccsch}(c x))}{3 e^2} - \frac{2 d (a + b \operatorname{arccsch}(c x)) \sqrt{e x + d}}{e^2}$$

$$+\frac{8 b d^2 \operatorname{EllipticPi}\left(\frac{\sqrt{1-x\sqrt{-c^2}}\,\sqrt{2}}{2}, 2, \sqrt{2}\sqrt{\frac{e}{d\sqrt{-c^2}+e}}\right)\sqrt{c^2x^2+1}\sqrt{\frac{(ex+d)\sqrt{-c^2}}{d\sqrt{-c^2}+e}}}{3 c e^2 x\sqrt{1+\frac{1}{c^2x^2}}\sqrt{ex+d}}$$

$$+\frac{4 b c \operatorname{EllipticE}\left(\frac{\sqrt{1-x\sqrt{-c^2}}\,\sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-c^2}}{c^2d-e\sqrt{-c^2}}}\right)\sqrt{ex+d}\sqrt{c^2x^2+1}}{3(-c^2)^{3/2} e x\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{c^2(ex+d)}{c^2d-e\sqrt{-c^2}}}}$$

$$-\frac{8 b c d \operatorname{EllipticF}\left(\frac{\sqrt{1-x\sqrt{-c^2}}\,\sqrt{2}}{2}, \sqrt{-\frac{2e\sqrt{-c^2}}{c^2d-e\sqrt{-c^2}}}\right)\sqrt{c^2x^2+1}\sqrt{\frac{c^2(ex+d)}{c^2d-e\sqrt{-c^2}}}}{3(-c^2)^{3/2} e x\sqrt{1+\frac{1}{c^2x^2}}\sqrt{ex+d}}$$

Result(type 4, 867 leaves):

$$\frac{1}{e^2}\left(2\left(a\left(\frac{(ex+d)^{3/2}}{3}-d\sqrt{ex+d}\right)+b\left(\frac{\operatorname{arccsch}(cx)\,(ex+d)^{3/2}}{3}-\operatorname{arccsch}(cx)\,d\sqrt{ex+d}\right.\right.\right.$$

$$-\left(2\sqrt{-\frac{I(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{I(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}}\left(2\,I\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}, \sqrt{-\frac{2Icde-c^2}{c^2d^2+}}\right.\right.\right.$$

$$-\operatorname{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}, \sqrt{-\frac{2Icde-c^2d^2+e^2}{c^2d^2+e^2}}\right)c^2d^2-\operatorname{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}, \sqrt{-\frac{2Icde-c^2d^2+e^2}{c^2d^2+e^2}}\right)c^2d^2$$

$$-2\,I\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}, \frac{c^2d^2+e^2}{(Ie+dc)cd}, \frac{\sqrt{-\frac{(Ie-dc)c}{c^2d^2+e^2}}}{\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}}\right)cde+2\operatorname{EllipticPi}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}, \frac{c^2d^2+e^2}{(Ie+dc)cd},\right.$$

$$\left.\frac{\sqrt{-\frac{(Ie-dc)c}{c^2d^2+e^2}}}{\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}}\right)c^2d^2+\text{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}},\sqrt{-\frac{2Icde-c^2d^2+e^2}{c^2d^2+e^2}}\right)e^2-\text{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}},\right.$$

$$\left.\left.\left.\sqrt{-\frac{2Icde-c^2d^2+e^2}{c^2d^2+e^2}}\right)e^2\right)\right)\Bigg/\left(\left(3c^2\sqrt{\frac{(ex+d)^2c^2-2(ex+d)c^2d+c^2d^2+e^2}{c^2x^2e^2}}\,x\sqrt{\frac{(Ie+dc)c}{c^2d^2+e^2}}\,(Ie-dc)\right)\right)\right)$$

Problem 20: Result more than twice size of optimal antiderivative.

$$\int\frac{x^2(a+b\,\text{arccsch}(cx))}{(ex+d)^{3/2}}\,dx$$

Optimal(type 4, 430 leaves, 16 steps):

$$\frac{2(ex+d)^{3/2}(a+b\,\text{arccsch}(cx))}{3e^3}-\frac{2d^2(a+b\,\text{arccsch}(cx))}{e^3\sqrt{ex+d}}-\frac{4d(a+b\,\text{arccsch}(cx))\sqrt{ex+d}}{e^3}$$

$$+\frac{32bd^2\,\text{EllipticPi}\left(\frac{\sqrt{1-x\sqrt{-c^2}}\sqrt{2}}{2},2,\sqrt{2}\sqrt{\frac{e}{d\sqrt{-c^2}+e}}\right)\sqrt{c^2x^2+1}\sqrt{\frac{(ex+d)\sqrt{-c^2}}{d\sqrt{-c^2}+e}}}{3ce^3x\sqrt{1+\frac{1}{c^2x^2}}\sqrt{ex+d}}$$

$$+\frac{4bc\,\text{EllipticE}\left(\frac{\sqrt{1-x\sqrt{-c^2}}\sqrt{2}}{2},\sqrt{-\frac{2e\sqrt{-c^2}}{c^2d-e\sqrt{-c^2}}}\right)\sqrt{ex+d}\sqrt{c^2x^2+1}}{3(-c^2)^{3/2}e^2x\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{c^2(ex+d)}{c^2d-e\sqrt{-c^2}}}}$$

$$-\frac{20bcd\,\text{EllipticF}\left(\frac{\sqrt{1-x\sqrt{-c^2}}\sqrt{2}}{2},\sqrt{-\frac{2e\sqrt{-c^2}}{c^2d-e\sqrt{-c^2}}}\right)\sqrt{c^2x^2+1}\sqrt{\frac{c^2(ex+d)}{c^2d-e\sqrt{-c^2}}}}{3(-c^2)^{3/2}e^2x\sqrt{1+\frac{1}{c^2x^2}}\sqrt{ex+d}}$$

Result(type 4, 895 leaves):

$$\frac{1}{e^3}\Bigg(2\Bigg(a\left(\frac{(ex+d)^{3/2}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+b\Bigg(\frac{\operatorname{arccsch}(cx)\,(ex+d)^{3/2}}{3}-2\operatorname{arccsch}(cx)\,d\sqrt{ex+d}-\frac{\operatorname{arccsch}(cx)\,d^2}{\sqrt{ex+d}}$$

$$-\Bigg(2\sqrt{-\frac{\mathrm{I}(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{\mathrm{I}(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}}\Bigg(5\,\mathrm{I}\,\mathrm{EllipticF}\Bigg(\sqrt{ex+d}\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}},$$

$$\sqrt{-\frac{2\mathrm{I}cde-c^2d^2+e^2}{c^2d^2+e^2}}\Bigg)c^2d^2-\mathrm{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}},\sqrt{-\frac{2\mathrm{I}cde-c^2d^2+e^2}{c^2d^2+e^2}}\right)c^2d^2-8\,\mathrm{I}\,\mathrm{EllipticPi}\Bigg(\sqrt{ex+d}\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}},$$

$$\frac{c^2d^2+e^2}{(\mathrm{I}e+dc)cd},\frac{\sqrt{-\frac{(\mathrm{I}e-dc)c}{c^2d^2+e^2}}}{\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}}}\Bigg)cde+8\,\mathrm{EllipticPi}\Bigg(\sqrt{ex+d}\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}},\frac{c^2d^2+e^2}{(\mathrm{I}e+dc)cd},\frac{\sqrt{-\frac{(\mathrm{I}e-dc)c}{c^2d^2+e^2}}}{\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}}}\Bigg)c^2d^2$$

$$+\mathrm{EllipticF}\left(\sqrt{ex+d}\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}},\sqrt{-\frac{2\mathrm{I}cde-c^2d^2+e^2}{c^2d^2+e^2}}\right)e^2-\mathrm{EllipticE}\left(\sqrt{ex+d}\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}},\sqrt{-\frac{2\mathrm{I}cde-c^2d^2+e^2}{c^2d^2+e^2}}\right)e^2\Bigg)\Bigg)\Bigg/$$

$$\left(3c^2\sqrt{\frac{(ex+d)^2c^2-2(ex+d)c^2d+c^2d^2+e^2}{c^2x^2e^2}}\,x\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}}\,(\mathrm{I}e-dc)\right)\Bigg)\Bigg)\Bigg)$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int\frac{a+b\operatorname{arccsch}(cx)}{(ex+d)^{3/2}}\,dx$$

Optimal(type 4, 132 leaves, 6 steps):

$$-\frac{2\left(a+b\,\mathrm{arccsch}(cx)\right)}{e\sqrt{ex+d}}+\frac{4\,b\,\mathrm{EllipticPi}\left(\frac{\sqrt{1-x\sqrt{-c^2}}\,\sqrt{2}}{2},2,\sqrt{2}\,\sqrt{\frac{e}{d\sqrt{-c^2}+e}}\right)\sqrt{c^2x^2+1}\,\sqrt{\frac{(ex+d)\sqrt{-c^2}}{d\sqrt{-c^2}+e}}}{c\,e\,x\sqrt{1+\frac{1}{c^2x^2}}\,\sqrt{ex+d}}$$

Result(type 4, 327 leaves):

$$\frac{1}{e}\left(2\left(-\frac{a}{\sqrt{ex+d}}+b\left(-\frac{\mathrm{arccsch}(cx)}{\sqrt{ex+d}}\right.\right.\right.$$

$$+\frac{1}{c\sqrt{\frac{(ex+d)^2c^2-2(ex+d)c^2d+c^2d^2+e^2}{c^2x^2e^2}}\,x\,d\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}}}\left(2\sqrt{-\frac{\mathrm{I}(ex+d)ce+(ex+d)c^2d-c^2d^2-e^2}{c^2d^2+e^2}}\right.$$

$$\left.\left.\left.\left.\sqrt{\frac{\mathrm{I}(ex+d)ce-(ex+d)c^2d+c^2d^2+e^2}{c^2d^2+e^2}}\,\mathrm{EllipticPi}\left(\sqrt{ex+d}\,\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}},\frac{c^2d^2+e^2}{(\mathrm{I}e+dc)cd},\frac{\sqrt{-\frac{(\mathrm{I}e-dc)c}{c^2d^2+e^2}}}{\sqrt{\frac{(\mathrm{I}e+dc)c}{c^2d^2+e^2}}}\right)\right)\right)\right)\right)$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int\frac{x^2\left(a+b\,\mathrm{arccsch}(cx)\right)}{(ex+d)^{5/2}}\,dx$$

Optimal(type 4, 496 leaves, 25 steps):

$$-\frac{2d^2\left(a+b\,\mathrm{arccsch}(cx)\right)}{3e^3(ex+d)^{3/2}}+\frac{4d\left(a+b\,\mathrm{arccsch}(cx)\right)}{e^3\sqrt{ex+d}}-\frac{4bd\left(c^2x^2+1\right)}{3ce\left(c^2d^2+e^2\right)x\sqrt{1+\frac{1}{c^2x^2}}\,\sqrt{ex+d}}+\frac{2\left(a+b\,\mathrm{arccsch}(cx)\right)\sqrt{ex+d}}{e^3}$$

$$-\frac{32\,b\,d\,\mathrm{EllipticPi}\left(\frac{\sqrt{1-x\sqrt{-c^2}}\,\sqrt{2}}{2},2,\sqrt{2}\,\sqrt{\frac{e}{d\sqrt{-c^2}+e}}\right)\sqrt{c^2x^2+1}\,\sqrt{\frac{(ex+d)\sqrt{-c^2}}{d\sqrt{-c^2}+e}}}{3\,c\,e^3x\sqrt{1+\frac{1}{c^2x^2}}\,\sqrt{ex+d}}$$

$$+\frac{4 b d \,\mathrm{EllipticE}\left(\frac{\sqrt{1-x\sqrt{-c^2}}\,\sqrt{2}}{2}, \sqrt{-\frac{2 e\sqrt{-c^2}}{c^2 d - e\sqrt{-c^2}}}\right)\sqrt{-c^2}\,\sqrt{e x + d}\,\sqrt{c^2 x^2+1}}{3 c e^2 \left(c^2 d^2 + e^2\right) x \sqrt{1+\frac{1}{c^2 x^2}}\,\sqrt{\frac{c^2 (e x + d)}{c^2 d - e\sqrt{-c^2}}}}$$

$$+\frac{4 b c \,\mathrm{EllipticF}\left(\frac{\sqrt{1-x\sqrt{-c^2}}\,\sqrt{2}}{2}, \sqrt{-\frac{2 e\sqrt{-c^2}}{c^2 d - e\sqrt{-c^2}}}\right)\sqrt{c^2 x^2+1}\,\sqrt{\frac{c^2 (e x + d)}{c^2 d - e\sqrt{-c^2}}}}{\left(-c^2\right)^{3/2} e^2 x \sqrt{1+\frac{1}{c^2 x^2}}\,\sqrt{e x + d}}$$

Result(type ?, 2496 leaves): Display of huge result suppressed!

Problem 28: Unable to integrate problem.

$$\int \frac{a + b\,\mathrm{arccsch}(c x)}{e x^2 + d}\, dx$$

Optimal(type 4, 485 leaves, 19 steps):

$$\frac{(a + b\,\mathrm{arccsch}(c x))\ln\left(1 - \frac{c\left(\frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{-c^2 d + e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{(a + b\,\mathrm{arccsch}(c x))\ln\left(1 + \frac{c\left(\frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{-c^2 d + e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

$$+\frac{(a + b\,\mathrm{arccsch}(c x))\ln\left(1 - \frac{c\left(\frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{-c^2 d + e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{(a + b\,\mathrm{arccsch}(c x))\ln\left(1 + \frac{c\left(\frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{-c^2 d + e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

$$-\frac{b\,\mathrm{polylog}\left(2, -\frac{c\left(\frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{-c^2 d + e}}\right)}{2\sqrt{-d}\,\sqrt{e}} + \frac{b\,\mathrm{polylog}\left(2, \frac{c\left(\frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{-c^2 d + e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

$$-\frac{b\,\mathrm{polylog}\left(2, -\frac{c\left(\frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{-c^2 d + e}}\right)}{2\sqrt{-d}\,\sqrt{e}} + \frac{b\,\mathrm{polylog}\left(2, \frac{c\left(\frac{1}{c x}+\sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{-c^2 d + e}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Result(type 8, 20 leaves):

$$\int \frac{a + b\,\mathrm{arccsch}(c\,x)}{e\,x^2 + d}\,dx$$

Problem 29: Unable to integrate problem.

$$\int \frac{a + b\,\mathrm{arccsch}(c\,x)}{x\,(e\,x^2 + d)}\,dx$$

Optimal(type 4, 463 leaves, 19 steps):

$$\frac{(a + b\,\mathrm{arccsch}(c\,x))^2}{2\,b\,d} - \frac{(a + b\,\mathrm{arccsch}(c\,x))\ln\left(1 - \frac{c\left(\frac{1}{c\,x} + \sqrt{1 + \frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2\,d + e}}\right)}{2\,d} - \frac{(a + b\,\mathrm{arccsch}(c\,x))\ln\left(1 + \frac{c\left(\frac{1}{c\,x} + \sqrt{1 + \frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2\,d + e}}\right)}{2\,d}$$
$$- \frac{(a + b\,\mathrm{arccsch}(c\,x))\ln\left(1 - \frac{c\left(\frac{1}{c\,x} + \sqrt{1 + \frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{-c^2\,d + e}}\right)}{2\,d} - \frac{(a + b\,\mathrm{arccsch}(c\,x))\ln\left(1 + \frac{c\left(\frac{1}{c\,x} + \sqrt{1 + \frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{-c^2\,d + e}}\right)}{2\,d}$$
$$- \frac{b\,\mathrm{polylog}\left(2, -\frac{c\left(\frac{1}{c\,x} + \sqrt{1 + \frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2\,d + e}}\right)}{2\,d} - \frac{b\,\mathrm{polylog}\left(2, \frac{c\left(\frac{1}{c\,x} + \sqrt{1 + \frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2\,d + e}}\right)}{2\,d}$$
$$- \frac{b\,\mathrm{polylog}\left(2, -\frac{c\left(\frac{1}{c\,x} + \sqrt{1 + \frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{-c^2\,d + e}}\right)}{2\,d} - \frac{b\,\mathrm{polylog}\left(2, \frac{c\left(\frac{1}{c\,x} + \sqrt{1 + \frac{1}{c^2\,x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{-c^2\,d + e}}\right)}{2\,d}$$

Result(type 8, 23 leaves):

$$\int \frac{a + b\,\mathrm{arccsch}(c\,x)}{x\,(e\,x^2 + d)}\,dx$$

Problem 30: Unable to integrate problem.

$$\int \frac{a + b\,\mathrm{arccsch}(c\,x)}{x^2\,(e\,x^2 + d)}\,dx$$

Optimal(type 4, 524 leaves, 24 steps):

$$-\frac{a}{dx}-\frac{b\,\mathrm{arccsch}(cx)}{dx}+\frac{(a+b\,\mathrm{arccsch}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)\sqrt{e}}{2(-d)^{3/2}}$$

$$-\frac{(a+b\,\mathrm{arccsch}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)\sqrt{e}}{2(-d)^{3/2}}+\frac{(a+b\,\mathrm{arccsch}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)\sqrt{e}}{2(-d)^{3/2}}$$

$$-\frac{(a+b\,\mathrm{arccsch}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)\sqrt{e}}{2(-d)^{3/2}}-\frac{b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)\sqrt{e}}{2(-d)^{3/2}}$$

$$+\frac{b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)\sqrt{e}}{2(-d)^{3/2}}-\frac{b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)\sqrt{e}}{2(-d)^{3/2}}$$

$$+\frac{b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{-c^2d+e}}\right)\sqrt{e}}{2(-d)^{3/2}}+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d}$$

Result(type 8, 23 leaves):

$$\int\frac{a+b\,\mathrm{arccsch}(cx)}{x^2\left(ex^2+d\right)}\,dx$$

Problem 31: Result more than twice size of optimal antiderivative.

$$\int\frac{x\,(a+b\,\mathrm{arccsch}(cx))}{\left(ex^2+d\right)^3}\,dx$$

Optimal(type 3, 180 leaves, 8 steps):

$$\frac{-a-b\,\mathrm{arccsch}(cx)}{4e\left(ex^2+d\right)^2}+\frac{bcx\arctan\left(\sqrt{-c^2x^2-1}\right)}{4d^2e\sqrt{-c^2x^2}}+\frac{bc\left(3c^2d-2e\right)x\,\mathrm{arctanh}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8d^2\left(c^2d-e\right)^{3/2}\sqrt{e}\sqrt{-c^2x^2}}+\frac{bcx\sqrt{-c^2x^2-1}}{8d\left(c^2d-e\right)\left(ex^2+d\right)\sqrt{-c^2x^2}}$$

Result(type 3, 1883 leaves):

$$
-\frac{c^4 a}{4 e (c^2 e x^2 + c^2 d)^2} - \frac{c^4 b \operatorname{arccsch}(c x)}{4 e (c^2 e x^2 + c^2 d)^2} - \frac{c^3 b \sqrt{c^2 x^2 + 1}\, x \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2 x^2 + 1}}\right) e}{4 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}\, d (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}
$$

$$
- \frac{c^3 b \sqrt{c^2 x^2 + 1} \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2 x^2 + 1}}\right)}{4 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}\, x (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}
$$

$$
+ \frac{3 c^3 b \sqrt{c^2 x^2 + 1}\, x \ln\left(-\frac{2\left(\sqrt{-\frac{c^2 d - e}{e}} \sqrt{c^2 x^2 + 1}\, e + \sqrt{-c^2 d e}\, c x + e\right)}{-c e x + \sqrt{-c^2 d e}}\right) e}{16 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}\, d \sqrt{-\frac{c^2 d - e}{e}}\, (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}
$$

$$
+ \frac{3 c^3 b \sqrt{c^2 x^2 + 1} \ln\left(-\frac{2\left(\sqrt{-\frac{c^2 d - e}{e}} \sqrt{c^2 x^2 + 1}\, e + \sqrt{-c^2 d e}\, c x + e\right)}{-c e x + \sqrt{-c^2 d e}}\right)}{16 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}\, x \sqrt{-\frac{c^2 d - e}{e}}\, (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}
$$

$$
+ \frac{3 c^3 b \sqrt{c^2 x^2 + 1}\, x \ln\left(\frac{2\left(\sqrt{-\frac{c^2 d - e}{e}} \sqrt{c^2 x^2 + 1}\, e - \sqrt{-c^2 d e}\, c x + e\right)}{c e x + \sqrt{-c^2 d e}}\right) e}{16 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}\, d \sqrt{-\frac{c^2 d - e}{e}}\, (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}
$$

$$
+ \frac{3 c^3 b \sqrt{c^2 x^2 + 1} \ln\left(\frac{2\left(\sqrt{-\frac{c^2 d - e}{e}} \sqrt{c^2 x^2 + 1}\, e - \sqrt{-c^2 d e}\, c x + e\right)}{c e x + \sqrt{-c^2 d e}}\right)}{16 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}\, x \sqrt{-\frac{c^2 d - e}{e}}\, (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}
$$

$$
+ \frac{c b \sqrt{c^2 x^2 + 1}\, x \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2 x^2 + 1}}\right) e^2}{4 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}\, d^2 (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)} + \frac{c b \sqrt{c^2 x^2 + 1} \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2 x^2 + 1}}\right) e}{4 \sqrt{\frac{c^2 x^2 + 1}{c^2 x^2}}\, x d (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}
$$

$$- \frac{c^3 b x e}{8 \sqrt{\frac{c^2 x^2+1}{c^2 x^2}}\, d\, (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)} - \frac{c b e}{8 \sqrt{\frac{c^2 x^2+1}{c^2 x^2}}\, x d\, (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}$$

$$- \frac{c b \sqrt{c^2 x^2+1}\, x \ln\left(-\frac{2 \left(\sqrt{-\frac{c^2 d - e}{e}} \sqrt{c^2 x^2+1}\, e + \sqrt{-c^2 d e}\, c x + e\right)}{-c e x + \sqrt{-c^2 d e}}\right) e^2}{8 \sqrt{\frac{c^2 x^2+1}{c^2 x^2}}\, d^2 \sqrt{-\frac{c^2 d - e}{e}}\, (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}$$

$$- \frac{c b \sqrt{c^2 x^2+1} \ln\left(-\frac{2 \left(\sqrt{-\frac{c^2 d - e}{e}} \sqrt{c^2 x^2+1}\, e + \sqrt{-c^2 d e}\, c x + e\right)}{-c e x + \sqrt{-c^2 d e}}\right) e}{8 \sqrt{\frac{c^2 x^2+1}{c^2 x^2}}\, x d \sqrt{-\frac{c^2 d - e}{e}}\, (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}$$

$$- \frac{c b \sqrt{c^2 x^2+1}\, x \ln\left(\frac{2 \left(\sqrt{-\frac{c^2 d - e}{e}} \sqrt{c^2 x^2+1}\, e - \sqrt{-c^2 d e}\, c x + e\right)}{c e x + \sqrt{-c^2 d e}}\right) e^2}{8 \sqrt{\frac{c^2 x^2+1}{c^2 x^2}}\, d^2 \sqrt{-\frac{c^2 d - e}{e}}\, (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}$$

$$- \frac{c b \sqrt{c^2 x^2+1} \ln\left(\frac{2 \left(\sqrt{-\frac{c^2 d - e}{e}} \sqrt{c^2 x^2+1}\, e - \sqrt{-c^2 d e}\, c x + e\right)}{c e x + \sqrt{-c^2 d e}}\right) e}{8 \sqrt{\frac{c^2 x^2+1}{c^2 x^2}}\, x d \sqrt{-\frac{c^2 d - e}{e}}\, (c^2 d - e) \left(-c e x + \sqrt{-c^2 d e}\right) \left(c e x + \sqrt{-c^2 d e}\right)}$$

Problem 32: Unable to integrate problem.

$$\int \frac{a + b \operatorname{arccsch}(c x)}{(e x^2 + d)^3}\, dx$$

Optimal(type 4, 984 leaves, 81 steps):

$$\frac{b\, e \operatorname{arctanh}\left(\frac{c^2 d - \frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d - e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right)}{16\, d^{5/2}\left(c^2 d - e\right)^{3/2}} + \frac{b\, e \operatorname{arctanh}\left(\frac{c^2 d + \frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d - e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right)}{16\, d^{5/2}\left(c^2 d - e\right)^{3/2}} + \frac{5\, b \operatorname{arctanh}\left(\frac{c^2 d - \frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d - e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right)}{16\, d^{5/2}\sqrt{c^2 d - e}}$$

$$+ \frac{5\, b \operatorname{arctanh}\left(\frac{c^2 d + \frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d - e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right)}{16\, d^{5/2}\sqrt{c^2 d - e}} + \frac{3\,\left(a + b \operatorname{arccsch}(c\,x)\right)\ln\left(1 - \frac{c\left(\frac{1}{c\,x} + \sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right)}{16\,(-d)^{5/2}\sqrt{e}}$$

$$- \frac{3\,\left(a + b \operatorname{arccsch}(c\,x)\right)\ln\left(1 + \frac{c\left(\frac{1}{c\,x} + \sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right)}{16\,(-d)^{5/2}\sqrt{e}} + \frac{3\,\left(a + b \operatorname{arccsch}(c\,x)\right)\ln\left(1 - \frac{c\left(\frac{1}{c\,x} + \sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right)}{16\,(-d)^{5/2}\sqrt{e}}$$

$$- \frac{3\,\left(a + b \operatorname{arccsch}(c\,x)\right)\ln\left(1 + \frac{c\left(\frac{1}{c\,x} + \sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right)}{16\,(-d)^{5/2}\sqrt{e}} - \frac{3\, b \operatorname{polylog}\left(2, -\frac{c\left(\frac{1}{c\,x} + \sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right)}{16\,(-d)^{5/2}\sqrt{e}}$$

$$+ \frac{3\, b \operatorname{polylog}\left(2, \frac{c\left(\frac{1}{c\,x} + \sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{-c^2 d + e}}\right)}{16\,(-d)^{5/2}\sqrt{e}} - \frac{3\, b \operatorname{polylog}\left(2, -\frac{c\left(\frac{1}{c\,x} + \sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right)}{16\,(-d)^{5/2}\sqrt{e}}$$

$$+ \frac{3\, b \operatorname{polylog}\left(2, \frac{c\left(\frac{1}{c\,x} + \sqrt{1+\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{-c^2 d + e}}\right)}{16\,(-d)^{5/2}\sqrt{e}} + \frac{\left(a + b \operatorname{arccsch}(c\,x)\right)\sqrt{e}}{16\,(-d)^{3/2}\left(-\frac{d}{x} + \sqrt{-d}\,\sqrt{e}\right)^2} - \frac{5\,\left(a + b \operatorname{arccsch}(c\,x)\right)}{16\, d^2\left(-\frac{d}{x} + \sqrt{-d}\,\sqrt{e}\right)}$$

$$- \frac{\left(a + b \operatorname{arccsch}(c\,x)\right)\sqrt{e}}{16\,(-d)^{3/2}\left(\frac{d}{x} + \sqrt{-d}\,\sqrt{e}\right)^2} + \frac{5\,\left(a + b \operatorname{arccsch}(c\,x)\right)}{16\, d^2\left(\frac{d}{x} + \sqrt{-d}\,\sqrt{e}\right)} - \frac{b\, c\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}}{16\,(-d)^{3/2}\left(c^2 d - e\right)\left(-\frac{d}{x} + \sqrt{-d}\,\sqrt{e}\right)}$$

$$-\frac{b c\sqrt{e}\sqrt{1+\frac{1}{c^2 x^2}}}{16(-d)^{3/2}(c^2 d-e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

Result(type 8, 20 leaves):

$$\int \frac{a+b\,\mathrm{arccsch}(c x)}{(e x^2+d)^3}\,dx$$

Problem 33: Unable to integrate problem.

$$\int x^5 (a+b\,\mathrm{arccsch}(c x))\sqrt{e x^2+d}\,dx$$

Optimal(type 3, 353 leaves, 12 steps):

$$\frac{d^2 (e x^2+d)^{3/2}(a+b\,\mathrm{arccsch}(c x))}{3 e^3}-\frac{2 d (e x^2+d)^{5/2}(a+b\,\mathrm{arccsch}(c x))}{5 e^3}+\frac{(e x^2+d)^{7/2}(a+b\,\mathrm{arccsch}(c x))}{7 e^3}$$
$$+\frac{b(105 d^3 c^6+35 c^4 d^2 e+63 c^2 d e^2-75 e^3)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-c^2 x^2-1}}{c\sqrt{e x^2+d}}\right)}{1680 c^6 e^{5/2}\sqrt{-c^2 x^2}}+\frac{8 b c d^{7/2} x\arctan\left(\frac{\sqrt{e x^2+d}}{\sqrt{d}\sqrt{-c^2 x^2-1}}\right)}{105 e^3\sqrt{-c^2 x^2}}$$
$$-\frac{b(29 c^2 d+25 e)\,x(e x^2+d)^{3/2}\sqrt{-c^2 x^2-1}}{840 c^3 e^2\sqrt{-c^2 x^2}}+\frac{b x (e x^2+d)^{5/2}\sqrt{-c^2 x^2-1}}{42 c e^2\sqrt{-c^2 x^2}}-\frac{b(23 c^4 d^2-12 c^2 d e-75 e^2)\,x\sqrt{-c^2 x^2-1}\sqrt{e x^2+d}}{1680 c^5 e^2\sqrt{-c^2 x^2}}$$

Result(type 8, 23 leaves):

$$\int x^5 (a+b\,\mathrm{arccsch}(c x))\sqrt{e x^2+d}\,dx$$

Problem 34: Unable to integrate problem.

$$\int x^3 (a+b\,\mathrm{arccsch}(c x))\sqrt{e x^2+d}\,dx$$

Optimal(type 3, 254 leaves, 11 steps):

$$-\frac{d (e x^2+d)^{3/2}(a+b\,\mathrm{arccsch}(c x))}{3 e^2}+\frac{(e x^2+d)^{5/2}(a+b\,\mathrm{arccsch}(c x))}{5 e^2}-\frac{b(15 c^4 d^2+10 c^2 d e-9 e^2)\,x\arctan\left(\frac{\sqrt{e}\sqrt{-c^2 x^2-1}}{c\sqrt{e x^2+d}}\right)}{120 c^4 e^{3/2}\sqrt{-c^2 x^2}}$$
$$-\frac{2 b c d^{5/2} x\arctan\left(\frac{\sqrt{e x^2+d}}{\sqrt{d}\sqrt{-c^2 x^2-1}}\right)}{15 e^2\sqrt{-c^2 x^2}}+\frac{b x (e x^2+d)^{3/2}\sqrt{-c^2 x^2-1}}{20 c e\sqrt{-c^2 x^2}}+\frac{b(c^2 d-9 e)\,x\sqrt{-c^2 x^2-1}\sqrt{e x^2+d}}{120 c^3 e\sqrt{-c^2 x^2}}$$

Result(type 8, 23 leaves):

$$\int x^3 \left(a + b \operatorname{arccsch}(c x)\right) \sqrt{e x^2 + d}\, dx$$

Problem 35: Unable to integrate problem.

$$\int x \left(a + b \operatorname{arccsch}(c x)\right) \sqrt{e x^2 + d}\, dx$$

Optimal(type 3, 167 leaves, 9 steps):

$$\frac{\left(e x^2 + d\right)^{3/2} \left(a + b \operatorname{arccsch}(c x)\right)}{3 e} + \frac{b c d^{3/2} x \arctan\left(\frac{\sqrt{e x^2 + d}}{\sqrt{d}\, \sqrt{-c^2 x^2 - 1}}\right)}{3 e \sqrt{-c^2 x^2}} + \frac{b \left(3 c^2 d - e\right) x \arctan\left(\frac{\sqrt{e}\, \sqrt{-c^2 x^2 - 1}}{c \sqrt{e x^2 + d}}\right)}{6 c^2 \sqrt{e}\, \sqrt{-c^2 x^2}} + \frac{b x \sqrt{-c^2 x^2 - 1}\, \sqrt{e x^2 + d}}{6 c \sqrt{-c^2 x^2}}$$

Result(type 8, 21 leaves):

$$\int x \left(a + b \operatorname{arccsch}(c x)\right) \sqrt{e x^2 + d}\, dx$$

Problem 38: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{arccsch}(c x)\right) \sqrt{e x^2 + d}}{x^4}\, dx$$

Optimal(type 4, 413 leaves, 8 steps):

$$-\frac{\left(e x^2 + d\right)^{3/2} \left(a + b \operatorname{arccsch}(c x)\right)}{3 d x^3} - \frac{2 b c^3 \left(c^2 d - 2 e\right) x^2 \sqrt{e x^2 + d}}{9 d \sqrt{-c^2 x^2}\, \sqrt{-c^2 x^2 - 1}} - \frac{2 b c \left(c^2 d - 2 e\right) \sqrt{-c^2 x^2 - 1}\, \sqrt{e x^2 + d}}{9 d \sqrt{-c^2 x^2}} + \frac{b c \sqrt{-c^2 x^2 - 1}\, \sqrt{e x^2 + d}}{9 x^2 \sqrt{-c^2 x^2}}$$

$$+ \frac{2 b c^2 \left(c^2 d - 2 e\right) x \sqrt{\frac{1}{c^2 x^2 + 1}}\, \sqrt{c^2 x^2 + 1}\, \operatorname{EllipticE}\left(\frac{c x}{\sqrt{c^2 x^2 + 1}}, \sqrt{1 - \frac{e}{c^2 d}}\right) \sqrt{e x^2 + d}}{9 d \sqrt{-c^2 x^2}\, \sqrt{-c^2 x^2 - 1}\, \sqrt{\frac{e x^2 + d}{d \left(c^2 x^2 + 1\right)}}}$$

$$- \frac{b \left(c^2 d - 3 e\right) e x \sqrt{\frac{1}{c^2 x^2 + 1}}\, \sqrt{c^2 x^2 + 1}\, \operatorname{EllipticF}\left(\frac{c x}{\sqrt{c^2 x^2 + 1}}, \sqrt{1 - \frac{e}{c^2 d}}\right) \sqrt{e x^2 + d}}{9 d^2 \sqrt{-c^2 x^2}\, \sqrt{-c^2 x^2 - 1}\, \sqrt{\frac{e x^2 + d}{d \left(c^2 x^2 + 1\right)}}}$$

Result(type 8, 23 leaves):

$$\int \frac{\left(a + b \operatorname{arccsch}(c x)\right) \sqrt{e x^2 + d}}{x^4}\, dx$$

Problem 42: Unable to integrate problem.

$$\int \frac{x^5 (a + b \operatorname{arccsch}(c x))}{(e x^2 + d)^{3/2}} \, dx$$

Optimal(type 3, 216 leaves, 10 steps):

$$\frac{(e x^2 + d)^{3/2} (a + b \operatorname{arccsch}(c x))}{3 e^3} - \frac{b (9 c^2 d + e) x \arctan\left(\frac{\sqrt{e} \sqrt{-c^2 x^2 - 1}}{c \sqrt{e x^2 + d}}\right)}{6 c^2 e^{5/2} \sqrt{-c^2 x^2}} - \frac{8 b c d^{3/2} x \arctan\left(\frac{\sqrt{e x^2 + d}}{\sqrt{d} \sqrt{-c^2 x^2 - 1}}\right)}{3 e^3 \sqrt{-c^2 x^2}} - \frac{d^2 (a + b \operatorname{arccsch}(c x))}{e^3 \sqrt{e x^2 + d}}$$
$$- \frac{2 d (a + b \operatorname{arccsch}(c x)) \sqrt{e x^2 + d}}{e^3} + \frac{b x \sqrt{-c^2 x^2 - 1} \sqrt{e x^2 + d}}{6 c e^2 \sqrt{-c^2 x^2}}$$

Result(type 8, 23 leaves):

$$\int \frac{x^5 (a + b \operatorname{arccsch}(c x))}{(e x^2 + d)^{3/2}} \, dx$$

Problem 43: Unable to integrate problem.

$$\int \frac{x^3 (a + b \operatorname{arccsch}(c x))}{(e x^2 + d)^{3/2}} \, dx$$

Optimal(type 3, 136 leaves, 9 steps):

$$\frac{b x \arctan\left(\frac{\sqrt{e} \sqrt{-c^2 x^2 - 1}}{c \sqrt{e x^2 + d}}\right)}{e^{3/2} \sqrt{-c^2 x^2}} + \frac{2 b c x \arctan\left(\frac{\sqrt{e x^2 + d}}{\sqrt{d} \sqrt{-c^2 x^2 - 1}}\right) \sqrt{d}}{e^2 \sqrt{-c^2 x^2}} + \frac{d (a + b \operatorname{arccsch}(c x))}{e^2 \sqrt{e x^2 + d}} + \frac{(a + b \operatorname{arccsch}(c x)) \sqrt{e x^2 + d}}{e^2}$$

Result(type 8, 23 leaves):

$$\int \frac{x^3 (a + b \operatorname{arccsch}(c x))}{(e x^2 + d)^{3/2}} \, dx$$

Problem 45: Unable to integrate problem.

$$\int \frac{a + b \operatorname{arccsch}(c x)}{x^2 (e x^2 + d)^{3/2}} \, dx$$

Optimal(type 4, 363 leaves, 7 steps):

$$\frac{-a - b \operatorname{arccsch}(c x)}{d x \sqrt{e x^2 + d}} - \frac{2 e x (a + b \operatorname{arccsch}(c x))}{d^2 \sqrt{e x^2 + d}} + \frac{b c^3 x^2 \sqrt{e x^2 + d}}{d^2 \sqrt{-c^2 x^2} \sqrt{-c^2 x^2 - 1}} + \frac{b c \sqrt{-c^2 x^2 - 1} \sqrt{e x^2 + d}}{d^2 \sqrt{-c^2 x^2}}$$

$$-\frac{b c^2 x \sqrt{\frac{1}{c^2 x^2+1}}\, \sqrt{c^2 x^2+1}\, \mathrm{EllipticE}\left(\frac{c x}{\sqrt{c^2 x^2+1}}, \sqrt{1-\frac{e}{c^2 d}}\right) \sqrt{e x^2+d}}{d^2 \sqrt{-c^2 x^2}\, \sqrt{-c^2 x^2-1}\, \sqrt{\frac{e x^2+d}{d\left(c^2 x^2+1\right)}}}$$

$$+\frac{2 b e x \sqrt{\frac{1}{c^2 x^2+1}}\, \sqrt{c^2 x^2+1}\, \mathrm{EllipticF}\left(\frac{c x}{\sqrt{c^2 x^2+1}}, \sqrt{1-\frac{e}{c^2 d}}\right) \sqrt{e x^2+d}}{d^3 \sqrt{-c^2 x^2}\, \sqrt{-c^2 x^2-1}\, \sqrt{\frac{e x^2+d}{d\left(c^2 x^2+1\right)}}}$$

Result(type 8, 23 leaves):

$$\int \frac{a+b \operatorname{arccsch}(c x)}{x^2 \left(e x^2+d\right)^{3/2}}\, dx$$

Problem 46: Unable to integrate problem.

$$\int \frac{x^5 (a+b \operatorname{arccsch}(c x))}{\left(e x^2+d\right)^{5/2}}\, dx$$

Optimal(type 3, 213 leaves, 10 steps):

$$-\frac{d^2 (a+b \operatorname{arccsch}(c x))}{3 e^3 \left(e x^2+d\right)^{3/2}}+\frac{b x \arctan\left(\frac{\sqrt{e}\, \sqrt{-c^2 x^2-1}}{c \sqrt{e x^2+d}}\right)}{e^{5/2} \sqrt{-c^2 x^2}}+\frac{8 b c x \arctan\left(\frac{\sqrt{e x^2+d}}{\sqrt{d}\, \sqrt{-c^2 x^2-1}}\right) \sqrt{d}}{3 e^3 \sqrt{-c^2 x^2}}+\frac{2 d (a+b \operatorname{arccsch}(c x))}{e^3 \sqrt{e x^2+d}}$$

$$+\frac{b c d x \sqrt{-c^2 x^2-1}}{3 \left(c^2 d-e\right) e^2 \sqrt{-c^2 x^2}\, \sqrt{e x^2+d}}+\frac{(a+b \operatorname{arccsch}(c x)) \sqrt{e x^2+d}}{e^3}$$

Result(type 8, 23 leaves):

$$\int \frac{x^5 (a+b \operatorname{arccsch}(c x))}{\left(e x^2+d\right)^{5/2}}\, dx$$

Problem 47: Unable to integrate problem.

$$\int (f x)^m \left(e x^2+d\right)^3 (a+b \operatorname{arccsch}(c x))\, dx$$

Optimal(type 5, 574 leaves, 6 steps):

$$\frac{d^3 (f x)^{1+m} (a+b \operatorname{arccsch}(c x))}{f (1+m)}+\frac{3 d^2 e (f x)^{3+m} (a+b \operatorname{arccsch}(c x))}{f^3 (3+m)}+\frac{3 d e^2 (f x)^{5+m} (a+b \operatorname{arccsch}(c x))}{f^5 (5+m)}+\frac{e^3 (f x)^{7+m} (a+b \operatorname{arccsch}(c x))}{f^7 (7+m)}$$

$$+\frac{b e\left(e^2\left(m^2+8 m+15\right)^2-3 c^2 d e(3+m)^2\left(m^2+13 m+42\right)+3 c^4 d^2\left(m^4+22 m^3+179 m^2+638 m+840\right)\right) x(f x)^{1+m} \sqrt{-c^2 x^2-1}}{c^5 f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m) \sqrt{-c^2 x^2}}$$

$$-\frac{b e^2\left(e(5+m)^2-3 c^2 d\left(m^2+13 m+42\right)\right) x(f x)^{3+m} \sqrt{-c^2 x^2-1}}{c^3 f^3(4+m)(5+m)(6+m)(7+m) \sqrt{-c^2 x^2}}+\frac{b e^3 x(f x)^{5+m} \sqrt{-c^2 x^2-1}}{c f^5(6+m)(7+m) \sqrt{-c^2 x^2}}$$

$$-\frac{1}{c^5 f(1+m)(2+m)(4+m)(6+m) \sqrt{-c^2 x^2} \sqrt{-c^2 x^2-1}}\left(b\left(\frac{c^6 d^3(2+m)(4+m)(6+m)}{1+m}\right.\right.$$

$$\left.-\frac{e(1+m)\left(e^2\left(m^2+8 m+15\right)^2-3 c^2 d e(3+m)^2\left(m^2+13 m+42\right)+3 c^4 d^2\left(m^4+22 m^3+179 m^2+638 m+840\right)\right)}{(3+m)(5+m)(7+m)}\right)$$

$$\left.x(f x)^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-c^2 x^2\right) \sqrt{c^2 x^2+1}\right)$$

Result(type 8, 25 leaves):

$$\int(f x)^m\left(e x^2+d\right)^3(a+b \operatorname{arccsch}(c x)) \, dx$$

Problem 48: Unable to integrate problem.

$$\int(f x)^m\left(e x^2+d\right)(a+b \operatorname{arccsch}(c x)) \, dx$$

Optimal(type 5, 206 leaves, 5 steps):

$$\frac{d(f x)^{1+m}(a+b \operatorname{arccsch}(c x))}{f(1+m)}+\frac{e(f x)^{3+m}(a+b \operatorname{arccsch}(c x))}{f^3(3+m)}+\frac{b e x(f x)^{1+m} \sqrt{-c^2 x^2-1}}{c f\left(m^2+5 m+6\right) \sqrt{-c^2 x^2}}$$

$$+\frac{b\left(e(1+m)^2-c^2 d(2+m)(3+m)\right) x(f x)^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2}+\frac{m}{2}\right],-c^2 x^2\right) \sqrt{c^2 x^2+1}}{c f(1+m)^2(2+m)(3+m) \sqrt{-c^2 x^2} \sqrt{-c^2 x^2-1}}$$

Result(type 8, 23 leaves):

$$\int(f x)^m\left(e x^2+d\right)(a+b \operatorname{arccsch}(c x)) \, dx$$

Test results for the 23 problems in "7.6.2 Inverse hyperbolic cosecant functions.txt"

Problem 3: Unable to integrate problem.

$$\int(f x+e)^3(a+b \operatorname{arccsch}(d x+c))^2 \, dx$$

Optimal(type 4, 551 leaves, 20 steps):

$$\frac{b^2 f^2(-c f+e d) x}{d^3}+\frac{b^2 f^3(d x+c)^2}{12 d^4}-\frac{(-c f+e d)^4(a+b \operatorname{arccsch}(d x+c))^2}{4 d^4 f}+\frac{(f x+e)^4(a+b \operatorname{arccsch}(d x+c))^2}{4 f}$$

$$- \frac{2 b f^2 (-cf+ed) (a+b\,\mathrm{arccsch}(dx+c))\,\mathrm{arctanh}\left(\frac{1}{dx+c}+\sqrt{1+\frac{1}{(dx+c)^2}}\right)}{d^4}$$

$$+ \frac{4 b (-cf+ed)^3 (a+b\,\mathrm{arccsch}(dx+c))\,\mathrm{arctanh}\left(\frac{1}{dx+c}+\sqrt{1+\frac{1}{(dx+c)^2}}\right)}{d^4} - \frac{b^2 f^3 \ln(dx+c)}{3 d^4} + \frac{3 b^2 f (-cf+ed)^2 \ln(dx+c)}{d^4}$$

$$- \frac{b^2 f^2 (-cf+ed)\,\mathrm{polylog}\left(2, -\frac{1}{dx+c}-\sqrt{1+\frac{1}{(dx+c)^2}}\right)}{d^4} + \frac{2 b^2 (-cf+ed)^3\,\mathrm{polylog}\left(2, -\frac{1}{dx+c}-\sqrt{1+\frac{1}{(dx+c)^2}}\right)}{d^4}$$

$$+ \frac{b^2 f^2 (-cf+ed)\,\mathrm{polylog}\left(2, \frac{1}{dx+c}+\sqrt{1+\frac{1}{(dx+c)^2}}\right)}{d^4} - \frac{2 b^2 (-cf+ed)^3\,\mathrm{polylog}\left(2, \frac{1}{dx+c}+\sqrt{1+\frac{1}{(dx+c)^2}}\right)}{d^4}$$

$$- \frac{b f^3 (dx+c) (a+b\,\mathrm{arccsch}(dx+c))\sqrt{1+\frac{1}{(dx+c)^2}}}{3 d^4} + \frac{3 b f (-cf+ed)^2 (dx+c) (a+b\,\mathrm{arccsch}(dx+c))\sqrt{1+\frac{1}{(dx+c)^2}}}{d^4}$$

$$+ \frac{b f^2 (-cf+ed) (dx+c)^2 (a+b\,\mathrm{arccsch}(dx+c))\sqrt{1+\frac{1}{(dx+c)^2}}}{d^4} + \frac{b f^3 (dx+c)^3 (a+b\,\mathrm{arccsch}(dx+c))\sqrt{1+\frac{1}{(dx+c)^2}}}{6 d^4}$$

Result(type 8, 22 leaves):

$$\int (fx+e)^3 (a+b\,\mathrm{arccsch}(dx+c))^2\, dx$$

Problem 4: Unable to integrate problem.

$$\int (a+b\,\mathrm{arccsch}(dx+c))^2\, dx$$

Optimal(type 4, 120 leaves, 8 steps):

$$\frac{(dx+c) (a+b\,\mathrm{arccsch}(dx+c))^2}{d} + \frac{4 b (a+b\,\mathrm{arccsch}(dx+c))\,\mathrm{arctanh}\left(\frac{1}{dx+c}+\sqrt{1+\frac{1}{(dx+c)^2}}\right)}{d}$$

$$+ \frac{2 b^2\,\mathrm{polylog}\left(2, -\frac{1}{dx+c}-\sqrt{1+\frac{1}{(dx+c)^2}}\right)}{d} - \frac{2 b^2\,\mathrm{polylog}\left(2, \frac{1}{dx+c}+\sqrt{1+\frac{1}{(dx+c)^2}}\right)}{d}$$

Result(type 8, 14 leaves):

$$\int (a+b\,\mathrm{arccsch}(dx+c))^2\, dx$$

Problem 8: Unable to integrate problem.

$$\int \frac{\operatorname{arccsch}(\sqrt{x})}{x}\, dx$$

Optimal(type 4, 48 leaves, 7 steps):

$$\operatorname{arccsch}(\sqrt{x})^2 - 2\operatorname{arccsch}(\sqrt{x}) \ln\left(1 - \left(\frac{1}{\sqrt{x}} + \sqrt{1 + \frac{1}{x}}\right)^2\right) - \operatorname{polylog}\left(2, \left(\frac{1}{\sqrt{x}} + \sqrt{1 + \frac{1}{x}}\right)^2\right)$$

Result(type 8, 10 leaves):

$$\int \frac{\operatorname{arccsch}(\sqrt{x})}{x}\, dx$$

Problem 10: Unable to integrate problem.

$$\int \frac{\operatorname{arccsch}(a\,x^n)}{x}\, dx$$

Optimal(type 4, 87 leaves, 7 steps):

$$\frac{\operatorname{arccsch}(a\,x^n)^2}{2\,n} - \frac{\operatorname{arccsch}(a\,x^n)\ln\left(1 - \left(\frac{1}{a\,x^n} + \sqrt{1 + \frac{1}{a^2\,(x^n)^2}}\right)^2\right)}{n} - \frac{\operatorname{polylog}\left(2, \left(\frac{1}{a\,x^n} + \sqrt{1 + \frac{1}{a^2\,(x^n)^2}}\right)^2\right)}{2\,n}$$

Result(type 8, 12 leaves):

$$\int \frac{\operatorname{arccsch}(a\,x^n)}{x}\, dx$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{\frac{1}{a\,x} + \sqrt{1 + \frac{1}{a^2\,x^2}}}{x^4}\, dx$$

Optimal(type 3, 53 leaves, 6 steps):

$$-\frac{1}{4\,a\,x^4} + \frac{a^3\operatorname{arccsch}(a\,x)}{8} - \frac{\sqrt{1 + \frac{1}{a^2\,x^2}}}{4\,x^3} - \frac{a^2\sqrt{1 + \frac{1}{a^2\,x^2}}}{8\,x}$$

Result(type 3, 172 leaves):

$$\frac{\sqrt{\frac{a^2x^2+1}{a^2x^2}}\,a^2\left(\left(\frac{a^2x^2+1}{a^2}\right)^{3/2}\sqrt{\frac{1}{a^2}}\,x^2a^2-\sqrt{\frac{a^2x^2+1}{a^2}}\,\sqrt{\frac{1}{a^2}}\,x^4a^2+\ln\left(\frac{2\left(\sqrt{\frac{1}{a^2}}\,\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+1\right)}{x\,a^2}\right)x^4-2\left(\frac{a^2x^2+1}{a^2}\right)^{3/2}\sqrt{\frac{1}{a^2}}\right)}{8x^3\sqrt{\frac{a^2x^2+1}{a^2}}\,\sqrt{\frac{1}{a^2}}}$$

$$-\frac{1}{4\,a\,x^4}$$

Problem 13: Result more than twice size of optimal antiderivative.

$$\int\left(\frac{1}{a\,x^2}+\sqrt{1+\frac{1}{a^2x^4}}\right)x^3\,dx$$

Optimal(type 3, 42 leaves, 6 steps):

$$\frac{x^2}{2\,a}+\frac{\operatorname{arctanh}\left(\sqrt{1+\frac{1}{a^2x^4}}\right)}{4\,a^2}+\frac{x^4\sqrt{1+\frac{1}{a^2x^4}}}{4}$$

Result(type 3, 93 leaves):

$$\frac{\sqrt{\frac{a^2x^4+1}{a^2x^4}}\,x^2\left(x^2\sqrt{\frac{a^2x^4+1}{a^2}}\,a^2+\ln\left(x^2+\sqrt{\frac{a^2x^4+1}{a^2}}\right)\right)}{4\sqrt{\frac{a^2x^4+1}{a^2}}\,a^2}+\frac{x^2}{2\,a}$$

Problem 14: Result more than twice size of optimal antiderivative.

$$\int\left(\frac{1}{a\,x^2}+\sqrt{1+\frac{1}{a^2x^4}}\right)x\,dx$$

Optimal(type 3, 34 leaves, 6 steps):

$$-\frac{\operatorname{arccsch}(a\,x^2)}{2\,a}+\frac{\ln(x)}{a}+\frac{x^2\sqrt{1+\frac{1}{a^2x^4}}}{2}$$

Result(type 3, 115 leaves):

$$\frac{\sqrt{\frac{a^2 x^4+1}{a^2 x^4}}\, x^2 \left(\sqrt{\frac{1}{a^2}}\, \sqrt{\frac{a^2 x^4+1}{a^2}}\, a^2 - \ln\left(\frac{2\left(\sqrt{\frac{1}{a^2}}\, \sqrt{\frac{a^2 x^4+1}{a^2}}\, a^2+1\right)}{x^2 a^2}\right)\right)}{2\sqrt{\frac{1}{a^2}}\, \sqrt{\frac{a^2 x^4+1}{a^2}}\, a^2} + \frac{\ln(x)}{a}$$

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{\frac{1}{a x^2} + \sqrt{1+\frac{1}{a^2 x^4}}}{x^3}\, dx$$

Optimal(type 3, 34 leaves, 6 steps):

$$-\frac{1}{4 a x^4} - \frac{a \operatorname{arccsch}(a x^2)}{4} - \frac{\sqrt{1+\frac{1}{a^2 x^4}}}{4 x^2}$$

Result(type 3, 113 leaves):

$$-\frac{\sqrt{\frac{a^2 x^4+1}{a^2 x^4}} \left(\ln\left(\frac{2\left(\sqrt{\frac{1}{a^2}}\, \sqrt{\frac{a^2 x^4+1}{a^2}}\, a^2+1\right)}{x^2 a^2}\right) x^4 + \sqrt{\frac{1}{a^2}}\, \sqrt{\frac{a^2 x^4+1}{a^2}}\right)}{4 x^2 \sqrt{\frac{a^2 x^4+1}{a^2}}\, \sqrt{\frac{1}{a^2}}} - \frac{1}{4 a x^4}$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(\frac{1}{a x} + \sqrt{1+\frac{1}{a^2 x^2}}\right)^2}{x^3}\, dx$$

Optimal(type 3, 59 leaves, 7 steps):

$$-\frac{1}{2 a^2 x^4} - \frac{1}{2 x^2} + \frac{a^2 \operatorname{arccsch}(a x)}{4} - \frac{\sqrt{1+\frac{1}{a^2 x^2}}}{2 a x^3} - \frac{a \sqrt{1+\frac{1}{a^2 x^2}}}{4 x}$$

Result(type 3, 175 leaves):

$$-\frac{1}{2 x^2} - \frac{1}{2 a^2 x^4}$$

$$+\frac{1}{4x^3\sqrt{\frac{a^2x^2+1}{a^2}}\sqrt{\frac{1}{a^2}}}\left(a\sqrt{\frac{a^2x^2+1}{a^2x^2}}\left(\left(\frac{a^2x^2+1}{a^2}\right)^{3/2}\sqrt{\frac{1}{a^2}}\,x^2a^2-\sqrt{\frac{a^2x^2+1}{a^2}}\sqrt{\frac{1}{a^2}}\,x^4a^2\right.\right.$$

$$\left.\left.+\ln\left(\frac{2\left(\sqrt{\frac{1}{a^2}}\sqrt{\frac{a^2x^2+1}{a^2}}\,a^2+1\right)}{x\,a^2}\right)x^4-2\left(\frac{a^2x^2+1}{a^2}\right)^{3/2}\sqrt{\frac{1}{a^2}}\right)\right)$$

```
Problem 19: Unable to integrate problem.
```

$$\int\frac{\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)(dx)^m}{c^2x^2+1}\,dx$$

```
Optimal(type 5, 75 leaves, 4 steps):
```

$$-\frac{d\,(dx)^{-1+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2}-\frac{m}{2}\right],\left[\frac{3}{2}-\frac{m}{2}\right],-\frac{1}{c^2x^2}\right)}{c^2(1-m)}+\frac{(dx)^m\,\mathrm{hypergeom}\left(\left[1,\frac{m}{2}\right],\left[1+\frac{m}{2}\right],-c^2x^2\right)}{c\,m}$$

```
Result(type 8, 38 leaves):
```

$$\int\frac{\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)(dx)^m}{c^2x^2+1}\,dx$$

```
Problem 20: Result more than twice size of optimal antiderivative.
```

$$\int\frac{\left(\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}\right)x}{c^2x^2+1}\,dx$$

```
Optimal(type 3, 25 leaves, 5 steps):
```

$$\frac{\arctan(cx)}{c^2}+\frac{\mathrm{arctanh}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c^2}$$

```
Result(type 3, 84 leaves):
```

$$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)}{\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2}+\frac{\arctan(cx)}{c^2}$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int \frac{\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}}{c^2x^2+1}\,dx$$

Optimal(type 3, 31 leaves, 7 steps):

$$-\frac{\operatorname{arccsch}(cx)}{c}+\frac{\ln(x)}{c}-\frac{\ln\left(c^2x^2+1\right)}{2c}$$

Result(type 3, 171 leaves):

$$\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,x\left(\sqrt{\frac{1}{c^2}}\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2-\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\,c^2\sqrt{\frac{1}{c^2}}-\ln\left(\frac{2\left(\sqrt{\frac{1}{c^2}}\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2+1\right)}{x\,c^2}\right)\right)}{\sqrt{\frac{1}{c^2}}\sqrt{\frac{c^2x^2+1}{c^2}}\,c^2}-\frac{\ln\left(c^2x^2+1\right)}{2c}$$

$$+\frac{\ln(x)}{c}$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int \frac{\frac{1}{cx}+\sqrt{1+\frac{1}{c^2x^2}}}{x\left(c^2x^2+1\right)}\,dx$$

Optimal(type 3, 28 leaves, 4 steps):

$$-\frac{1}{cx}-\arctan(cx)-\sqrt{1+\frac{1}{c^2x^2}}$$

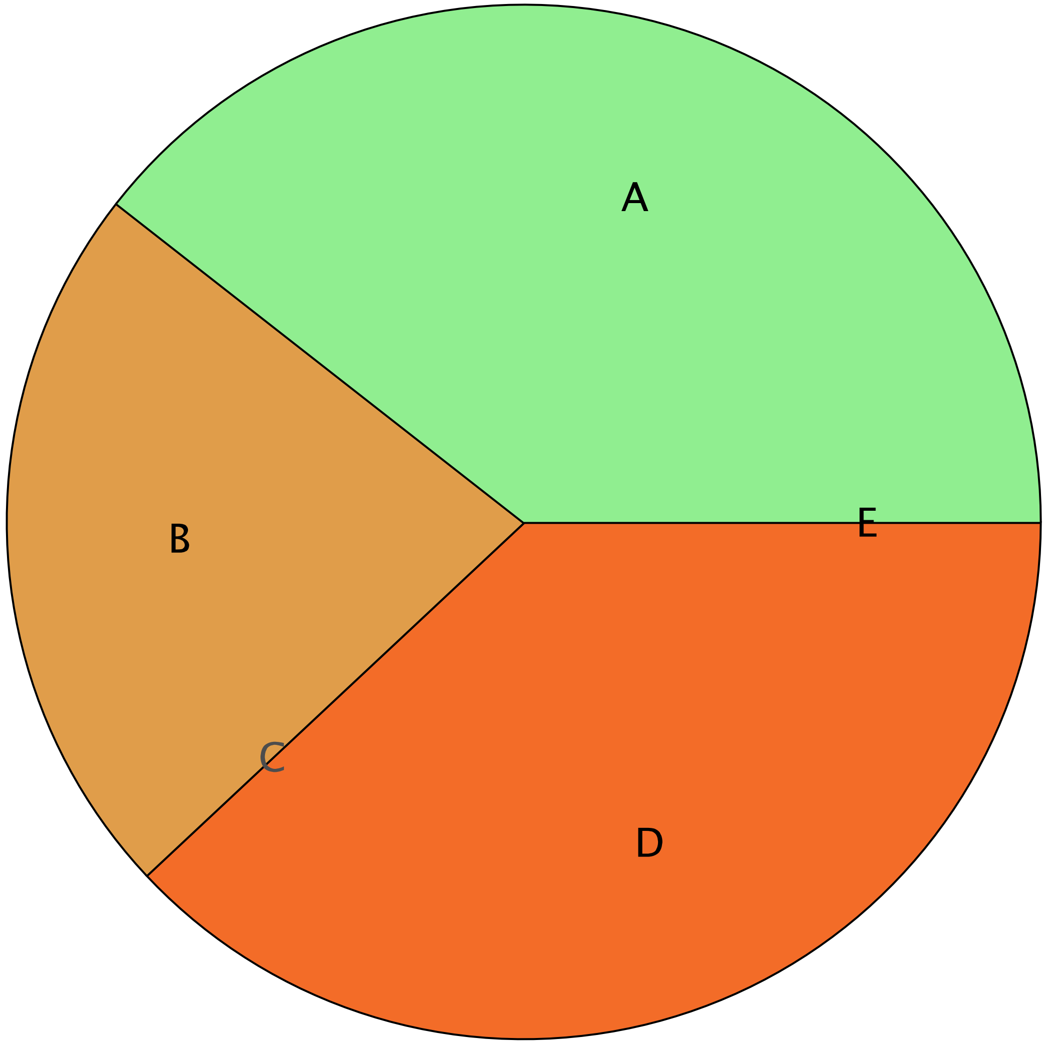
Result(type 3, 153 leaves):

$$-\frac{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(c^2\left(\frac{c^2x^2+1}{c^2}\right)^{3/2}-c^2x^2\sqrt{\frac{c^2x^2+1}{c^2}}+\ln\left(x+\sqrt{-\frac{\left(-c^2x+\sqrt{-c^2}\right)\left(c^2x+\sqrt{-c^2}\right)}{c^4}}\right)x-\ln\left(x+\sqrt{\frac{c^2x^2+1}{c^2}}\right)x\right)}{\sqrt{\frac{c^2x^2+1}{c^2}}}-\arctan(cx)$$

$$-\frac{1}{cx}$$

Summary of Integration Test Results

71 integration problems




```
A - 28 optimal antiderivatives
B - 16 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 27 unable to integrate problems
E - 0 integration timeouts
```